Algebra I (IMRA)

Subject: Mathematics
Grade: 09
Expectations: 56
Breakouts: 188

(a) Introduction.

1. The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

2. The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

3. In Algebra I, students will build on the knowledge and skills for mathematics in Grades 6-8, which provide a foundation in linear relationships, number and operations, and proportionality. Students will study linear, quadratic, and exponential functions and their related transformations, equations, and associated solutions. Students will connect functions and their associated solutions in both mathematical and real-world situations. Students will use technology to collect and explore data and analyze statistical relationships. In addition, students will study polynomials of degree one and two, radical expressions, sequences, and laws of exponents. Students will generate and solve linear systems with two equations and two variables and will create new functions through transformations.

4. Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(b) Knowledge and Skills Statements

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

   (A) apply mathematics to problems arising in everyday life, society, and the workplace;

      (i) apply mathematics to problems arising in everyday life

      (ii) apply mathematics to problems arising in society

      (iii) apply mathematics to problems arising in the workplace

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

  (i) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process

  (ii) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the reasonableness of the solution

(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

  (i) select tools, including real objects as appropriate, to solve problems

  (ii) select tools, including manipulatives as appropriate, to solve problems

  (iii) select tools, including paper and pencil as appropriate, to solve problems

  (iv) select tools, including technology as appropriate, to solve problems

  (v) select techniques, including mental math as appropriate, to solve problems

  (vi) select techniques including estimation as appropriate, to solve problems

  (vii) select techniques, including number sense as appropriate, to solve problems

(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

  (i) communicate mathematical ideas using multiple representations, including symbols as appropriate

  (ii) communicate mathematical ideas using multiple representations, including diagrams as appropriate

  (iii) communicate mathematical ideas using multiple representations, including graphs as appropriate

  (iv) communicate mathematical ideas using multiple representations, including language as appropriate

  (v) communicate mathematical reasoning using multiple representations, including symbols as appropriate

  (vi) communicate mathematical reasoning using multiple representations, including diagrams as appropriate

  (vii) communicate mathematical reasoning using multiple representations, including graphs as appropriate

  (viii) communicate mathematical reasoning using multiple representations, including language as appropriate

  (ix) communicate [mathematical ideas'] implications using multiple representations, including symbols as appropriate

  (x) communicate [mathematical ideas'] implications using multiple representations, including diagrams as appropriate

  (xi) communicate [mathematical ideas'] implications using multiple representations, including graphs as appropriate

  (xii) communicate [mathematical ideas'] implications using multiple representations, including language as appropriate

  (xiii) communicate [mathematical reasoning's] implications using multiple representations, including symbols as appropriate

<div style="margin-left: 2em;">

(xiv) communicate [mathematical reasoning's] implications using multiple representations, including diagrams as appropriate

(xv) communicate [mathematical reasoning's] implications using multiple representations, including graphs as appropriate

(xvi) communicate [mathematical reasoning's] implications using multiple representations, including language as appropriate

</div>

(E) create and use representations to organize, record, and communicate mathematical ideas;

(i) create representations to organize mathematical ideas

(ii) create representations to record mathematical ideas

(iii) create representations to communicate mathematical ideas

(iv) use representations to organize mathematical ideas

(v) use representations to record mathematical ideas

(vi) use representations to communicate mathematical ideas

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

(i) analyze mathematical relationships to connect mathematical ideas

(ii) analyze mathematical relationships to communicate mathematical ideas

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(i) display mathematical ideas using precise mathematical language in written or oral communication

(ii) display mathematical arguments using precise mathematical language in written or oral communication

(iii) explain mathematical ideas using precise mathematical language in written or oral communication

(iv) explain mathematical arguments using precise mathematical language in written or oral communication

(v) justify mathematical ideas using precise mathematical language in written or oral communication

(vi) justify mathematical arguments using precise mathematical language in written or oral communication

(2) Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:

(A) determine the domain and range of a linear function in mathematical problems; determine reasonable domain and range values for real-world situations, both continuous and discrete; and represent domain and range using inequalities;

(i) determine the domain of a linear function in mathematical problems

(ii) determine the range of a linear function in mathematical problems

(iii) determine reasonable domain values for real-world situations, both continuous and discrete

(iv) determine reasonable range values for real-world situations, both continuous and discrete

(v) represent domain using inequalities

(vi) represent range using inequalities

(B) write linear equations in two variables in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m (x - x_1)$, given one point and the slope and given two points

    (i)   write linear equations in two variables in various forms, including $y = mx + b$ given one point and the slope

    (ii)   write linear equations in two variables in various forms, including $y = mx + b$ given two points

    (iii)   write linear equations in two variables in various forms, including $Ax + By = C$, given one point and the slope

    (iv)   write linear equations in two variables in various forms, including $Ax + By = C$, given two points

    (v)   write linear equations in two variables in various forms, including $y - y_1 = m (x - x_1)$, given one point and the slope

    (vi)   write linear equations in two variables in various forms, including $y - y_1 = m (x - x_1)$, given two points

(C) write linear equations in two variables given a table of values, a graph, and a verbal description;

    (i)   write linear equations in two variables given a table of values

    (ii)   write linear equations in two variables given a graph

    (iii)   write linear equations in two variables given a verbal description

(D) write and solve equations involving direct variation;

    (i)   write equations involving direct variation

    (ii)   solve equations involving direct variation

(E) write the equation of a line that contains a given point and is parallel to a given line;

    (i)   write the equation of a line that contains a given point and is parallel to a given line

(F) write the equation of a line that contains a given point and is perpendicular to a given line;

    (i)   write the equation of a line that contains a given point and is perpendicular to a given line

(G) write an equation of a line that is parallel or perpendicular to the X or Y axis and determine whether the slope of the line is zero or undefined;

    (i)   write an equation of a line that is parallel or perpendicular to the X or Y axis

    (ii)   determine whether the slope of the line is zero or undefined

(H) write linear inequalities in two variables given a table of values, a graph, and a verbal description; and

    (i)   write linear inequalities in two variables given a table of values

    (ii)   write linear inequalities in two variables given a graph

    (iii)   write linear inequalities in two variables given a verbal description

(I) write systems of two linear equations given a table of values, a graph, and a verbal description.

    (i)   write systems of two linear equations given a table of values

    (ii)   write systems of two linear equations given a graph

    (iii)   write systems of two linear equations given a verbal description

(3) Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:

(A) determine the slope of a line given a table of values, a graph, two points on the line, and an equation written in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$

   (i)   determine the slope of a line given a table of values

   (ii)   determine the slope of a line given a graph

   (iii)   determine the slope of a line given two points on the line

   (iv)   determine the slope of a line given an equation written in various forms, including $y = mx + b$

   (v)   determine the slope of a line given an equation written in various forms, including $Ax + By = C$

   (vi)   determine the slope of a line given an equation written in various forms, including $y - y_1 = m(x - x_1)$

(B) calculate the rate of change of a linear function represented tabularly, graphically, or algebraically in context of mathematical and real-world problems;

   (i)   calculate the rate of change of a linear function represented tabularly, graphically, or algebraically in context of mathematical problems

   (ii)   calculate the rate of change of a linear function represented tabularly, graphically, or algebraically in context of real-world problems

(C) graph linear functions on the coordinate plane and identify key features, including x- intercept, y- intercept, zeros, and slope, in mathematical and real-world problems;

   (i)   graph linear functions on the coordinate plane in mathematical problems

   (ii)   graph linear functions on the coordinate plane in real-world problems

   (iii)   identify key features, including x- intercept in mathematical problems

   (iv)   identify key features, including x- intercept in real-world problems

   (v)   identify key features, including y- intercept in mathematical problems

   (vi)   identify key features, including y- intercept in real-world problems

   (vii)   identify key features, including zeros in mathematical problems

   (viii)   identify key features, including zeros in real-world problems

   (ix)   identify key features, including slope in mathematical problems

   (x)   identify key features, including slope in real-world problems

(D) graph the solution set of linear inequalities in two variables on the coordinate plane;

   (i)   graph the solution set of linear inequalities in two variables on the coordinate plane

(E) determine the effects on the graph of the parent function $f(x) = x$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of $a, b, c,$ and d

   (i)   determine the effects on the graph of the parent function $f(x) = x$ when $f(x)$ is replaced by $af(x)$ for specific values of a

   (ii)   determine the effects on the graph of the parent function $f(x) = x$ when $f(x)$ is replaced by $f(x) + d$ for specific values of d

        (iii)   determine the effects on the graph of the parent function $f(x) = x$ when $f(x)$ is replaced by $f(x - c)$ for specific values of c

        (iv)   determine the effects on the graph of the parent function $f(x) = x$ when $f(x)$ is replaced by $f(bx)$ for specific values of b

  (F) graph systems of two linear equations in two variables on the coordinate plane and determine the solutions if they exist;

        (i)   graph systems of two linear equations in two variables on the coordinate plane

        (ii)   determine the solutions if they exist

  (G) estimate graphically the solutions to systems of two linear equations with two variables in real-world problems; and

        (i)   estimate graphically the solutions to systems of two linear equations with two variables in real-world problems

  (H) graph the solution set of systems of two linear inequalities in two variables on the coordinate plane.

        (i)   graph the solution set of systems of two linear inequalities in two variables on the coordinate plane

(4) Linear functions, equations, and inequalities. The student applies the mathematical process standards to formulate statistical relationships and evaluate their reasonableness based on real-world data. The student is expected to:

  (A) calculate, using technology, the correlation coefficient between two quantitative variables and interpret this quantity as a measure of the strength of the linear association;

        (i)   calculate, using technology, the correlation coefficient between two quantitative variables

        (ii)   interpret this quantity as a measure of the strength of the linear association

  (B) compare and contrast association and causation in real-world problems; and

        (i)   compare and contrast association and causation in real-world problems

  (C) write, with and without technology, linear functions that provide a reasonable fit to data to estimate solutions and make predictions for real-world problems.

        (i)   write, with technology, linear functions that provide a reasonable fit to data to estimate solutions

        (ii)   write, without technology, linear functions that provide a reasonable fit to data to estimate solutions

        (iii)   make predictions for real-world problems

(5) Linear functions, equations, and inequalities. The student applies the mathematical process standards to solve, with and without technology, linear equations and evaluate the reasonableness of their solutions. The student is expected to:

  (A) solve linear equations in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides;

        (i)   solve linear equations in one variable, including those for which the application of the distributive property is necessary

        (ii)   solve linear equations in one variable, including those for which variables are included on both sides

  (B) solve linear inequalities in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides; and

        (i)   solve linear inequalities in one variable, including those for which the application of the distributive property is necessary

      (ii)   solve linear inequalities in one variable, including those for which variables are included on both sides

   (C)   solve systems of two linear equations with two variables for mathematical and real-world problems.

      (i)   solve systems of two linear equations with two variables for mathematical problems

      (ii)   solve systems of two linear equations with two variables for real-world problems

(6)   Quadratic functions and equations. The student applies the mathematical process standards when using properties of quadratic functions to write and represent in multiple ways, with and without technology, quadratic equations. The student is expected to:

   (A)   determine the domain and range of quadratic functions and represent the domain and range using inequalities;

      (i)   determine the domain of quadratic functions

      (ii)   determine the range of quadratic functions

      (iii)   represent the domain using inequalities

      (iv)   represent the range using inequalities

   (B)   write equations of quadratic functions given the vertex and another point on the graph, write the equation in vertex form $(f(x) = a(x - h)^2 + k)$, and rewrite the equation from vertex form to standard form $(f(x) = ax^2 + bx + c)$; and

      (i)   write equations of quadratic functions given the vertex and another point on the graph

      (ii)   write the equation [of quadratic functions] in vertex form $(f(x) = a(x - h)^2 + k)$

      (iii)   rewrite the equation [of quadratic functions] from vertex form to standard form $(f(x) = ax^2 + bx + c)$

   (C)   write quadratic functions when given real solutions and graphs of their related equations.

      (i)   write quadratic functions when given real solutions

      (ii)   write quadratic functions when given graphs of their related equations

(7)   Quadratic functions and equations. The student applies the mathematical process standards when using graphs of quadratic functions and their related transformations to represent in multiple ways and determine, with and without technology, the solutions to equations. The student is expected to:

   (A)   graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including x- intercept, y- intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry;

      (i)   graph quadratic functions on the coordinate plane

      (ii)   use the graph to identify key attributes, if possible, including x- intercept

      (iii)   use the graph to identify key attributes, if possible, including y- intercept

      (iv)   use the graph to identify key attributes, if possible, including zeros

      (v)   use the graph to identify key attributes, if possible, including maximum value

      (vi)   use the graph to identify key attributes, if possible, including minimum values

      (vii)   use the graph to identify key attributes, if possible, including vertex

      (viii)   use the graph to identify key attributes, if possible, including the equation of the axis of symmetry

   (B)   describe the relationship between the linear factors of quadratic expressions and the zeros of their associated quadratic functions; and

      (i)   describe the relationship between the linear factors of quadratic expressions and the zeros of their associated quadratic functions

(C) determine the effects on the graph of the parent function $f(x) = x^2$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c, and d

      (i)   determine the effects on the graph of the parent function $f(x) = x^2$ when $f(x)$ is replaced by $af(x)$ for specific values of a

      (ii)   determine the effects on the graph of the parent function $f(x) = x^2$ when $f(x)$ is replaced by $f(x) + d$ for specific values of d

      (iii)   determine the effects on the graph of the parent function $f(x) = x^2$ when $f(x)$ is replaced by $f(x - c)$ for specific values of c

      (iv)   determine the effects on the graph of the parent function $f(x) = x^2$ when f(x) is replaced by $f(bx)$ for specific values of b

(8) Quadratic functions and equations. The student applies the mathematical process standards to solve, with and without technology, quadratic equations and evaluate the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:

(A) solve quadratic equations having real solutions by factoring, taking square roots, completing the square, and applying the quadratic formula; and

      (i)   solve quadratic equations having real solutions by factoring

      (ii)   solve quadratic equations having real solutions by taking square roots

      (iii)   solve quadratic equations having real solutions by completing the square

      (iv)   solve quadratic equations having real solutions by applying the quadratic formula

(B) write, using technology, quadratic functions that provide a reasonable fit to data to estimate solutions and make predictions for real-world problems.

      (i)   write, using technology, quadratic functions that provide a reasonable fit to data to estimate solutions

      (ii)   write, using technology, quadratic functions that provide a reasonable fit to data to make predictions for real-world problems

(9) Exponential functions and equations. The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:

(A) determine the domain and range of exponential functions of the form $f(x) = ab^x$ and represent the domain and range using inequalities

      (i)   determine the domain of exponential functions of the form $f(x) = ab^x$

      (ii)   determine the range of exponential functions of the form $f(x) = ab^x$

      (iii)   represent the domain [of exponential functions of the form $f(x) = ab^x$] using inequalities

      (iv)   represent the range [of exponential functions of the form $f(x) = ab^x$] using inequalities

(B) interpret the meaning of the values of a and b in exponential functions of the form $f(x) = ab^x$ in real-world problems

(i)   interpret the meaning of the values of *a* exponential functions of the form $f(x) = ab^x$ in real-world problems

(ii)   interpret the meaning of the values of *b* in exponential functions of the form $f(x) = ab^x$ in real-world problems

(C) write exponential functions in the form $f(x) = ab^x$ (where *b* is a rational number) to describe problems arising from mathematical and real-world situations, including growth and decay

    (i)   write exponential functions in the form $f(x) = ab^x$ (where *b* is a rational number) to describe problems arising from mathematical situations, including growth

    (ii)   write exponential functions in the form $f(x) = ab^x$ (where *b* is a rational number) to describe problems arising from mathematical situations, including decay

    (iii)   write exponential functions in the form $f(x) = ab^x$ (where *b* is a rational number) to describe problems arising from real-world situations, including growth

    (iv)   write exponential functions in the form $f(x) = ab^x$ (where *b* is a rational number) to describe problems arising from real-world situations, including decay

(D) graph exponential functions that model growth and decay and identify key features, including *y*- intercept and asymptote, in mathematical and real-world problems; and

    (i)   graph exponential functions that model growth

    (ii)   graph exponential functions that model decay

    (iii)   identify key features, including *y*- intercept, in mathematical problems

    (iv)   identify key features, including *y*- intercept, in real-world problems

    (v)   identify key features, including asymptote, in mathematical problems

    (vi)   identify key features, including asymptote, in real-world problems

(E) write, using technology, exponential functions that provide a reasonable fit to data and make predictions for real-world problems.

    (i)   write, using technology, exponential functions that provide a reasonable fit to data

    (ii)   make predictions for real-world problems

(10) Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to rewrite in equivalent forms and perform operations on polynomial expressions. The student is expected to:

(A) add and subtract polynomials of degree one and degree two;

    (i)   add polynomials of degree one

    (ii)   add polynomials of degree two

    (iii)   add polynomials of degree one and two

    (iv)   subtract polynomials of degree one

    (v)   subtract polynomials of degree two

    (vi)   subtract polynomials of degree one [from] degree two

(B) multiply polynomials of degree one and degree two;

    (i)   multiply polynomials of degree one

       (ii)   multiply polynomials of degree two

       (iii)   multiply polynomials of degree one and two

(C) determine the quotient of a polynomial of degree one and polynomial of degree two when divided by a polynomial of degree one and polynomial of degree two when the degree of the divisor does not exceed the degree of the dividend;

       (i)   determine the quotient of a polynomial of degree one when divided by a polynomial of degree one

       (ii)   determine the quotient of a polynomial of degree two when divided by a polynomial of degree one

       (iii)   determine the quotient of a polynomial of degree two when divided by a polynomial of degree two

(D) rewrite polynomial expressions of degree one and degree two in equivalent forms using the distributive property;

       (i)   rewrite polynomial expressions of degree one in equivalent forms using the distributive property

       (ii)   rewrite polynomial expressions of degree two in equivalent forms using the distributive property

(E) factor, if possible, trinomials with real factors in the form $ax^2 + bx + c$, including perfect square trinomials of degree two; and

       (i)   factor, if possible, trinomials with real factors in the form $ax^2 + bx + c$, including perfect square trinomials of degree two

(F) decide if a binomial can be written as the difference of two squares and, if possible, use the structure of a difference of two squares to rewrite the binomial.

       (i)   decide if a binomial can be written as the difference of two squares

       (ii)   if [a binomial can be written as the difference of two squares], use the structure of a difference of two squares to rewrite the binomial

(11) Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to rewrite algebraic expressions into equivalent forms. The student is expected to:

(A) simplify numerical radical expressions involving square roots; and

       (i)   simplify numerical radical expressions involving square roots

(B) simplify numeric and algebraic expressions using the laws of exponents, including integral and rational exponents.

       (i)   simplify numeric expressions using the laws of exponents, including integral exponents

       (ii)   simplify numeric expressions using the laws of exponents, including rational exponents

       (iii)   simplify algebraic expressions using the laws of exponents, including integral exponents

       (iv)   simplify algebraic expressions using the laws of exponents, including rational exponents

(12) Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions. The student is expected to:

(A) decide whether relations represented verbally, tabularly, graphically, and symbolically define a function;

       (i)   decide whether relations represented verbally define a function

       (ii)   decide whether relations represented tabularly define a function

       (iii)   decide whether relations represented graphically define a function

       (iv)   decide whether relations represented symbolically define a function

(B) evaluate functions, expressed in function notation, given one or more elements in their domains;

    (i)   evaluate functions, expressed in function notation, given one or more elements in their domains

(C) identify terms of arithmetic and geometric sequences when the sequences are given in function form using recursive processes;

    (i)   identify terms of arithmetic sequences when the sequences are given in function form using recursive processes

    (ii)   identify terms of geometric sequences when the sequences are given in function form using recursive processes

(D) write a formula for the nth term of arithmetic and geometric sequences, given the value of several of their terms; and

    (i)   write a formula for the nth term of arithmetic sequences, given the value of several of their terms

    (ii)   write a formula for the nth term of geometric sequences, given the value of several of their terms

(E) solve mathematic and scientific formulas, and other literal equations, for a specified variable.

    (i)   solve mathematic formulas for a specified variable

    (ii)   solve scientific formulas for a specified variable

    (iii)   solve other literal equations for a specified variable